Intermediate Microeconomics

Part I: Welfare Economics

Session 4: Welfare

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INTRODUCTION

In **sessions 2-3**, we have defined the Pareto criterion and used it to evaluate if the market is efficient. Now, *is the Pareto criterion a satisfying tool for public policy?* There are some problems left apart:

- Taking a situation where the total amount of goods available is (for example) 100, both social states $(0, 100)$ and $(50, 50)$ are Pareto optima. However, some would find it unacceptable that an individual possesses all the goods (100), whereas the other individual has nothing (0)
- The Pareto criterion is silent on judging which of the social states $(0, 100)$ and $(50, 50)$ is better than the other
  - To take yet another (more striking) example, it cannot say if $(1, 1\ 000\ 000)$ is better or worse than $(999\ 998, 999\ 999)$!
- That is, it cannot judge that one social state is *fair*, so it seems like a *restrictive* normative criterion for public policy

Also, individuals' preference may be different (I may prefer x over y, you may prefer y over x):

- How do we aggregate individual preferences in order to have a *social* (or collective) preference?

INTRODUCTION

In **session 4**, we will be concerned about:

- Other normative criteria for policy evaluation
- The methods of aggregating individuals' preferences into a collective preference and their problems

INTRODUCTION

We will be concerned about the idea of a **welfare function:** a way to ”add together” different individuals’ utilities.

More generally, a welfare function provides a way to rank different distributions of utility among individuals.

Before we investigate the implications of this concept, it is worthwhile considering just how one might go about ”adding together” individuals’ preferences to construct some kind of ”social preferences”.

AGGREGATION OF PREFERENCES

Let us denote by **x** a particular allocation — a description of what every individual gets of every good (for example: for three individuals, **x** = (5, 3, 8) means that $u_A = 5; u_B = 3; u_C = 8$).

Then given two allocations, **x** and **y**, each individual can say whether or not he or she prefers **x** to **y**.

Given the preferences of all the individuals, we would like to have a way to "aggregate" them into one social preference.

If we know how all the individuals rank various allocations, we would like to be able to use this information to develop a social ranking of the various allocations.

AGGREGATION OF PREFERENCES: MAJORITY

One way to aggregate individual preferences is to use some kind of voting.

We could agree that **x** is "socially preferred" to **y** if a **majority** of the individuals prefer **x** to **y**.

However, there is a problem with this method: it may not generate a transitive social preference ordering.

AGGREGATION OF PREFERENCES: PAIRWISE MAJORITY

To show it, let's consider three individuals (A, B, C) who have the following preferences about three alternatives **(x, y, z)**:

A	B	C
x	y	z
y	z	x
z	x	y

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y	**z**	**x**
z	**x**	**y**

A majority of the people prefer **x** to **y**, a majority prefer **y** to **z**, and a majority prefer **z** to **x**. We have then: $x \succ y \succ z \succ x$!

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Problem: social preferences are not transitive. This information cannot tell us which outcome is preferred by society.

Pairwise voting does not always aggregate transitive individual preferences into a transitive social preference.

AGGREGATION OF PREFERENCES: PAIRWISE MAJORITY CONTEST

Let's try something else then. What if A, B, C decide to vote first on **x** *versus* **y**, then vote on the winner of this contest *versus* **z**?

A	B	C
x	y	z
y	z	x
z	x	y

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A	B	C
x	**y**	**z**
y	**z**	**x**
z	**x**	**y**

Contest 1: a majority prefer x to y, that is $(x \succ y) \times 2$ against $(y \succ x) \times 1$
Contest 2: the second contest is then between x and z, where $(z \succ x) \times 2$ against $(x \succ z) \times 1$, which means that z would be socially preferred

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Problem: how about if they start to vote on z *versus* x and then pit the winner of this vote against y? In this case, z would win the first contest and y the second one.

Which outcome society chooses will depend on the *order* in which the vote is taken. But there is no reason why x, y or z should come first.

AGGREGATION OF PREFERENCES: RANK-ORDER

Let's try something else then. Each individual ranks the alternatives according to his/her preferences and assigns a number that indicates its rank in his ordering.

For example: 1 = best alternative ; 2 = second best ; 3 = third best alternative.

We then sum up the scores of each alternative across the individuals to determine an aggregate score for each alternative and say that one outcome is socially preferred to another if it has a **lower** score.

Rank	A	B	C
1	x	y	z
2	y	z	x
3	z	x	y

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Rank	A	B	C
1	x	y	z
2	y	z	x
3	z	x	y

Problem: each gets 3 + 2 + 1 = 6 points, so rank-order voting is indecisive in this case as well!

AGGREGATION OF PREFERENCES: RANK-ORDER

What if we add another social state w, such that:

Rank	A	B	C
1	x	y	z
2	y	z	x
3	z	w	y
4	w	x	w

AGGREGATION OF PREFERENCES: RANK-ORDER

What if we add another social state w, such that:

Rank	A	B	C
1	x	y	z
2	y	z	x
3	z	w	y
4	w	x	w

Problem: we have x = 7, y = 6, z = 6 and w = 11, so we still cannot say anything on which social state wins (y *versus* z is a draw).

Even worse: one individual can intentionally lie about his/her ranking so that his/her most preferred alternative wins.

Assume you are *C*. Can you think of something which will make your top alternative (z) win?

AGGREGATION OF PREFERENCES: MANIPULATION

Assume that C is astute: he observes that his most preferred alternative (z) will not win (draw with y = 6), so he lies about his preferences.

He still puts z in top position but says that in fact, he prefers $z \succ w \succ x \succ y$:

Rank	A	B	C
1	x	y	z
2	y	z	w
3	z	w	x
4	w	x	y

We observe that if he lies, then x = 8, y = 7, z = 6 and w = 9, so **z wins!**

Note: Gibbard and Satterthwaite (1973, 1975) found that there does not exist a voting procedure for which individuals have never an incentive to strategically change their vote.

AGGREGATION OF PREFERENCES: MANIPULATION

The problem with both majority voting and rank-order voting is that their outcomes can be manipulated by astute individuals. We have seen that:

- Majority voting can be manipulated by changing the order on which things are voted so as to yield the desired outcome
- Rank-order voting can be manipulated by introducing new alternatives that change the final ranks of the relevant alternatives

So are there ways of aggregating preferences that are immune to this kind of manipulation?[1]

That is, are there ways to "add up" preferences that don't have the undesirable properties described above?

[1] Technically, we call a way to aggregate preferences a "social decision mechanism".

FINDING A DESIRABLE SOCIAL DECISION MECHANISM

Let's list some things that we would want our social decision mechanism to do:

1. Given any set of complete, reflexive, and transitive individual preferences, the social decision mechanism should result in social preferences that satisfy the same properties

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All three of these requirements seem eminently plausible. Yet it is hard to find a mechanism that satisfies all of them. In fact, Arrow (1951) has proved the following remarkable result:

Arrow's Impossibility Theorem. *If a social decision mechanism satisfies properties 1, 2, and 3, then it must be a dictatorship: all social rankings are the rankings of one individual.*

ARROWS' IMPOSSIBILITY THEOREM

A better way to summarise this theorem is that a social mechanism cannot satisfy *all* of these four properties at the same time:

1. **Unrestricted domain.** Given any set of complete, reflexive, and transitive individual preferences, the social decision mechanism should result in social preferences that satisfy the same properties
2. **Unanimity.** If everybody prefers alternative x to alternative y, then the social preferences should rank x ahead of y
3. **Independence of alternatives.** The preferences between x and y should depend only on how people rank x versus y, and not on how they rank other alternatives
4. **Non-dictatorship.** The social rankings cannot be exactly the same as *one* individual ranking
   - For example, if individuals have different preferences over alternatives, and if one individual states that $x \succ y \succ z$ then the social preference cannot be $x \succ y \succ z$

That is, Arrow showed (through a mathematical demonstration) that a social mechanism can only satisfy 3 out of these 4 properties, but not all of them!

ARROWS' IMPOSSIBILITY THEOREM

So if we want to find a way to aggregate individual preferences to form a social preference, we will have to **give up one of these properties** of a social decision mechanism described in Arrow's theorem:

1. **Unrestricted domain.** Given any set of complete, reflexive, and transitive individual preferences, the social decision mechanism should result in social preferences that satisfy the same properties
2. **Unanimity.** If everybody prefers alternative x to alternative y, then the social preferences should rank x ahead of y
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   - For example, if individuals have different preferences over alternatives, and if one individual states that $x \succ y \succ z$ then the social preference cannot be $x \succ y \succ z$

Find the one that you like the less: hard choice, right?

Social Welfare Functions

Note: A lot of researchers have tried to relax the conditions of Arrow's impossibility theorem, but there is no consensus on which are the desirable properties of a social decision mechanism.

(These debates involve *value* judgements, not judgements of *fact*).

- Although it is impossible to define social preferences very well, we can still define rules that tell us which outcome is best for society
- These rules would be incomplete, but still provide a good basis for public policy evaluation

In order to evaluate society, we can use a **social welfare function**: a function which assigns to each input (individuals' utilities) an output (the social utility).

Social Welfare Functions

In other words, a social welfare function (SWF) is simply a normative criterion applied to society as a whole.

To the question "what is best for society?", we can have different SWF which give us different results about the outcome of the society.

Let us denote by $W(u_1, ..., u_n)$ a social welfare function.

For example (and in the case that society is composed by two individuals):

- If $W(u_A, u_B) = u_A$, it means that society only cares about A's utility
- If $W(u_A, u_B) = 0.5u_A + 0.5u_B$, it means that society cares about each one's utility equally
- If $W(u_A, u_B) = 2u_A + u_B$, it means that society cares about A's utility twice as much as B's utility
- If $W(u_A, u_B) = u_A \times u_B$, it means that social utility highly depends on the interdependence between A's and B's utility
  - For example, if A's or B's utility = 0, then society's utility = 0
- And so on... (propose your own!)

Social Welfare Functions

More particularly, a SWF gives a way to rank different allocations that:

1. depends only on the individual preferences (and nothing else); and that
2. is an increasing function of each individual's utility
   - This means that if all individuals prefer x to y, then the social preference will prefer x to y; or
   - If an individual prefers x to y, and if none of the other individuals prefer y to x, then x should be regarded as socially preferable to y

These are the only two assumptions we will make about SWF! The rest is up to economists/philosophers to propose what they judge to be a "good" SWF.

Let's review some known SWF.

UTILITARIAN SWF

Utilitarianism (Bentham 1823; Mill 1863): we should aim for the greatest happiness of the greatest number.

According to a *utilitarian* criterion, one social state should be preferred to another if the **sum** of individuals' utilities is higher. Formally, if:

$$\sum_{i=1}^{n} u_i(\mathrm{x}) > \sum_{i=1}^{n} u_i(\mathrm{y})$$

Then x should be preferred to y ($\mathrm{x} \succ \mathrm{y}$). We have thus the following SWF:

$$W(u_1, \ldots, u_n) = \sum_{i=1}^{n} u_i$$

which states that social welfare depends on the sum of individuals' utilities.

UTILITARIAN WEIGHTED SWF

A slight generalisation of this form is the weighted-sum-of-utilities welfare function:

$$W(u_1, ..., u_n) = \sum_{i=1}^{n} a_i u_i$$

where the weights, $a_1, ..., a_n$, are supposed to be numbers indicating how important each individual's utility is to the overall social welfare.

Note: It is natural to take each a_i as being positive.

EGALITARIAN LIBERAL SWF

Another known social welfare function is the **minimax** or **Rawlsian social welfare function.**

Egalitarian liberalism (Rawls 1971): Rawls argues that justice as *fairness* is superior to the dominant tradition in modern political thought: *utilitarianism*.

One important condition of Rawls' principle of justice as fairness is that social and economic inequalities are to be to the **greatest benefit of the least-advantaged members of society**. Formally,

$$W(u_1, ..., u_n) = \min\{u_1, ..., u_n\}$$

This welfare function says that the social welfare of an allocation depends only on the welfare of the worst off individual: the person with the minimal utility.

COMMENTS ON THE SWF

- Each of these SWF is a possible way to compare individual utility functions
- Each of them represents different ethical judgments about the comparison between different individuals' welfares
- So there are no good and bad SWF! It really depends on what you judge to be the most important value for evaluating society
  - Some would provide arguments for utilitarianism (Harsanyi 1955)
  - Some would provide arguments for egalitarian liberalism (Rawls 1971)
  - Some would provide arguments for capabilities (Sen 1985)
  - And so on...

About the only restriction that we will place on the structure of the social welfare function is that it be increasing in each individual's utility:

- That is, if an individual prefers x to y, and if none of the other individuals prefer y to x, then x should be regarded as socially preferable to y

SOCIAL WELFARE MAXIMISATION

Once we have a social welfare function (choose the one you like!) we can examine the problem of welfare maximisation.

Just like in consumer theory where individuals aim at maximising their utility, if the goal of a policymaker is to maximise social welfare then he would have to maximise individuals' utilities according to the constraint of allocating *all* the goods to the individuals (otherwise there would be leftovers). Formally:

$$\max W(u_1(\mathrm{x}), ..., u_n(\mathrm{x}))$$

$$\text{such that: } \sum_{i=1}^{n} x_i^1 = X^1, ..., \sum_{i=1}^{n} x_i^k = X^k$$

- Where x_i^j indicates how much individual i has of good j, with n individuals and k goods
- Where $X^1, ..., X^k$ are the total amount of goods $1, ..., k$ to distribute among individuals

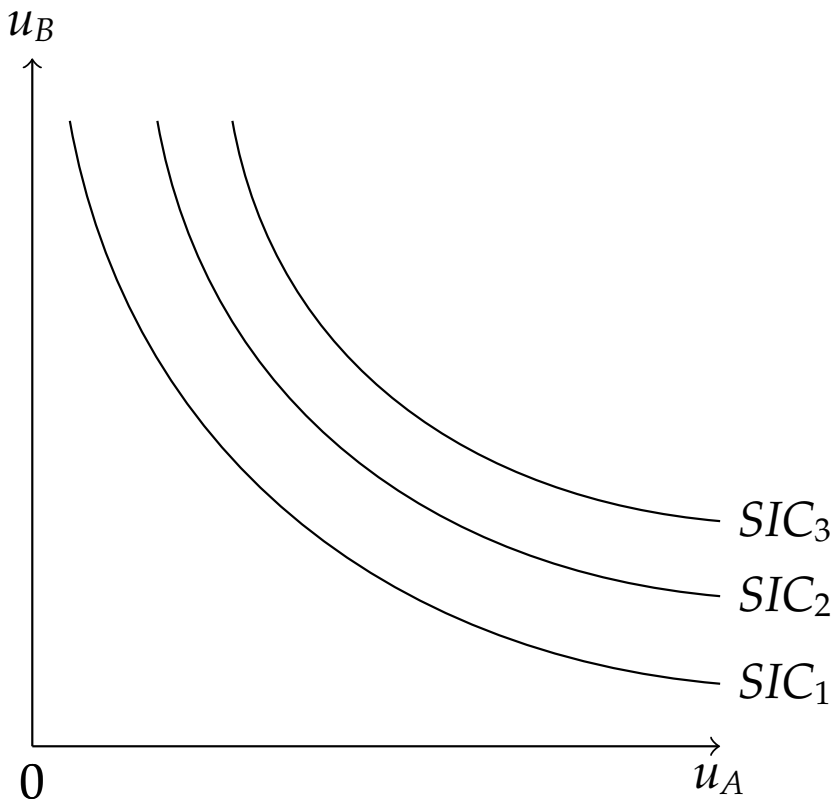
The policymaker (or social planner) aims at finding the feasible allocation that maximises social welfare.

SOCIAL INDIFFERENCE CURVES

- We can now represent graphically how optimal social states would look like according to some SWF with the use of *social indifference curves*
- Just like *individual* indifference curves represent the level of one's utility between two goods (x_1 and x_2), here *social* indifference curves represent the social utility between the utility of two individuals (u_A and u_B)
- Specifically, different SWF will lead to different social indifference curves

SOCIAL INDIFFERENCE CURVES

Assume that society is composed by two individuals: A and B.
If $W(u_A, u_B) = u_A \times u_B$, possible social indifference curves (SIC) could be:

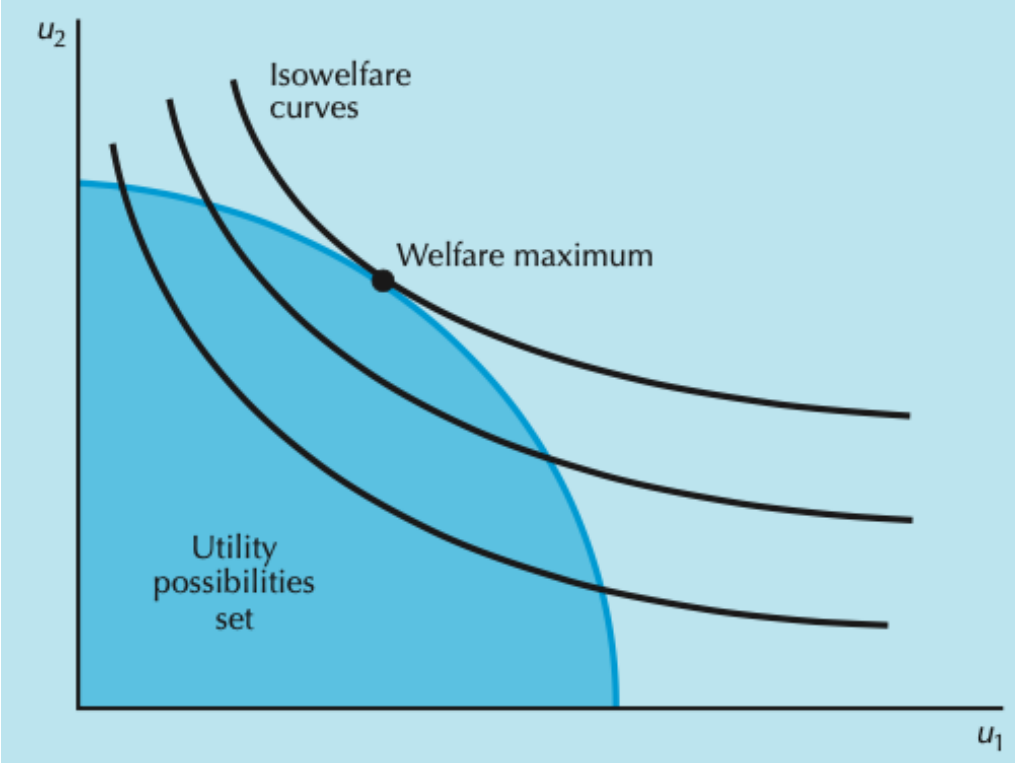


Where a higher social indifference curve represents a higher level of social utility.

SOCIAL INDIFFERENCE CURVES

Social welfare is maximised when the the social indifference curve is tangent to the social constraint, represented by the **utility possibilities set**.

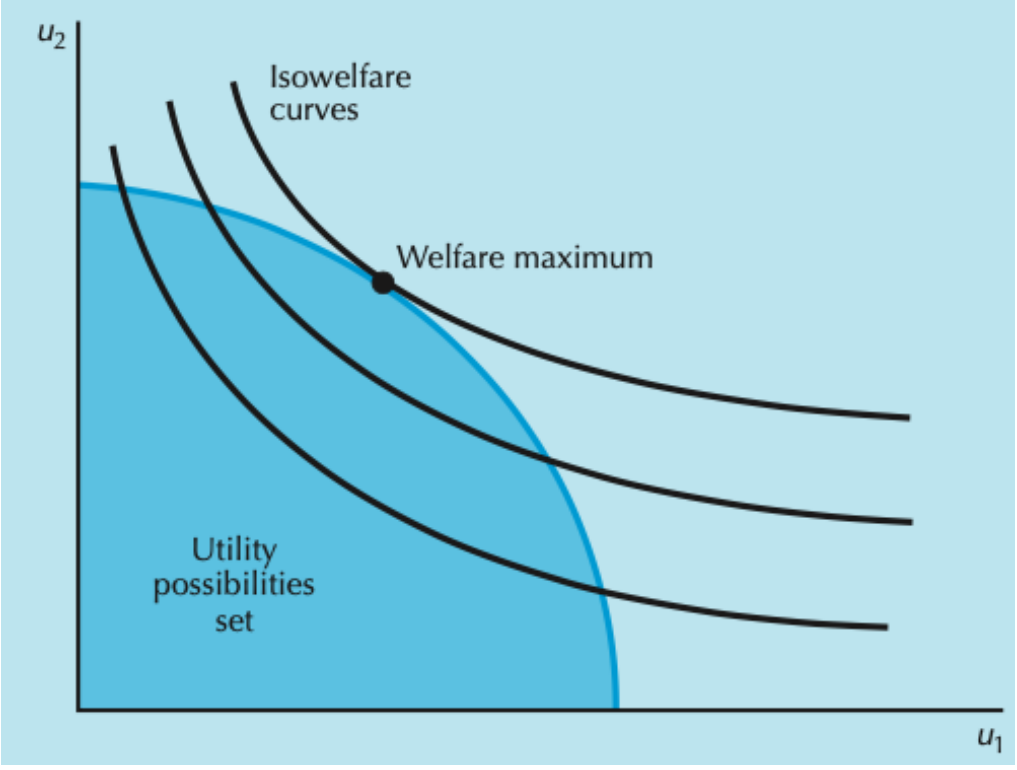
The utility possibilities set indicates all the possible utilities that two individuals can have.



SOCIAL INDIFFERENCE CURVES

The social indifference curves in this diagram are called **isowelfare curves** since they depict those distributions of utility that have constant welfare.

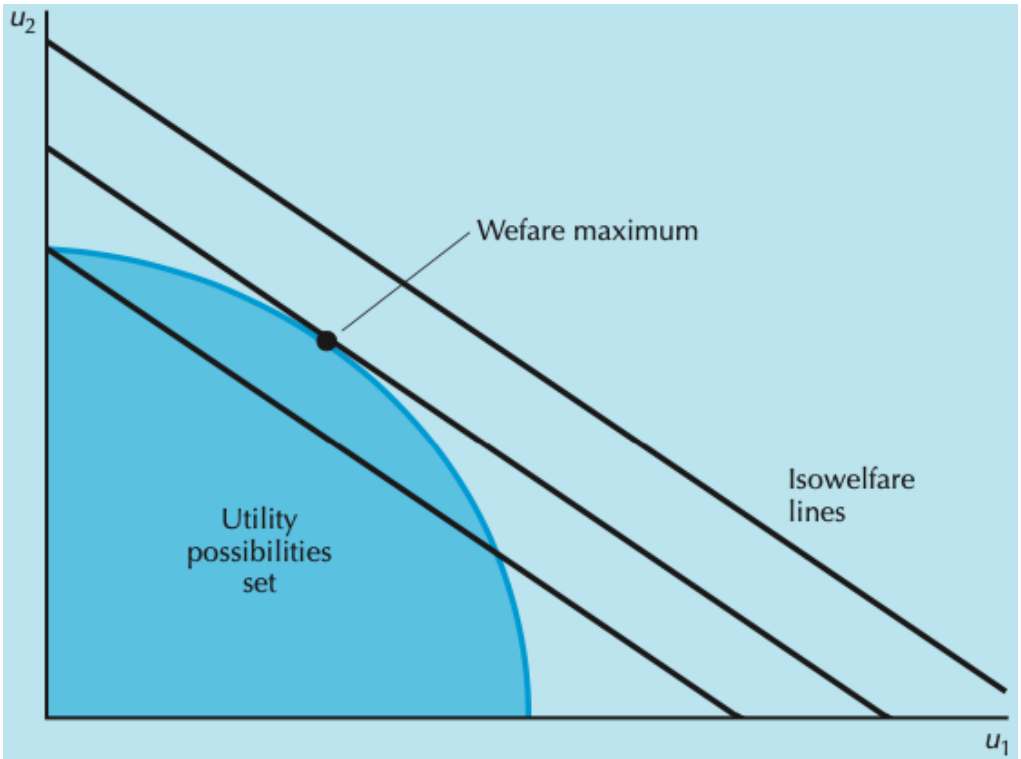
Note that the maximal welfare point is **Pareto efficient**: it occurs on the boundary of the utility possibilities set.



SOCIAL INDIFFERENCES CURVES: UTILITARIAN

If $W(u_A, u_B) = u_A + u_B$; or $W(u_A, u_B) = \alpha_A u_A + \alpha_B u_B$,

then possible social indifference curves (*SIC*) could be:

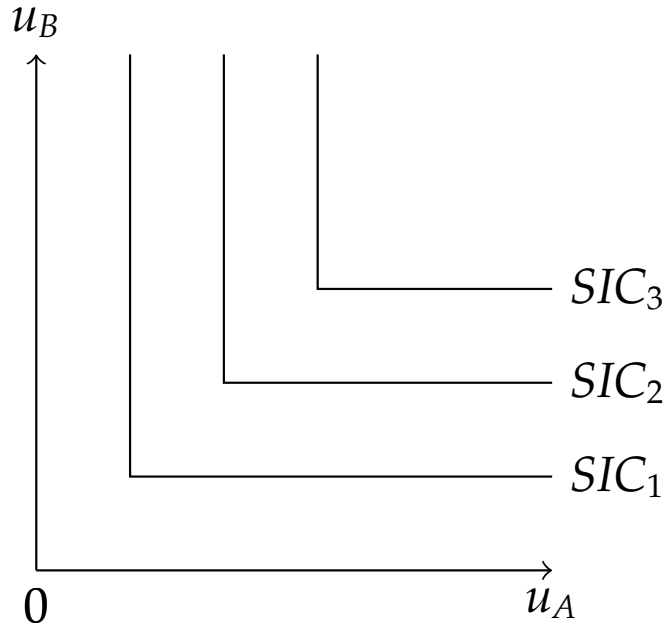


SOCIAL INDIFFERENCE CURVES: EGALITARIAN LIBERALISM

How would social indifference curves of an egalitarian liberal SWF would look like?

SOCIAL INDIFFERENCE CURVES: EGALITARIAN LIBERALISM

How would social indifference curves of an egalitarian liberal SWF would look like? Answer: like this:



- Social welfare remains the same when only one individual increases his/her utility
  - For example: (5,1) $\sim$ (1000,1)
- According to the egalitarian-liberal SCW, we need that both A and B increase their utility in order to judge that one social state is preferred to another
  - For example: (5,1) $\prec$ (1000,2)

A QUICK NOTE ON RAWLS' THEORY OF JUSTICE

So according to the egalitarian-liberal criterion, inequalities are justified only if they allow to make the worst off individuals better off.

In fact, Rawls (1971) proposed two principles of justice as *fairness*:

- **First Principle.** Each person has the same indefeasible claim to a fully adequate scheme of equal basic liberties, which scheme is compatible with the same scheme of liberties for all;
- **Second Principle.** Social and economic inequalities are to satisfy two conditions:
  - They are to be attached to offices and positions open to all under conditions of fair equality of opportunity;
  - They are to be to the greatest benefit of the least-advantaged members of society (the difference principle)

Such an *egalitarian* normative criterion proposed by Rawls is not the only one! There are many others, but this takes us out of the scope of economic theory.

SUMMARY

So what can we conclude about evaluating social well-being?

- That there is no social decision mechanism that satisfies some basic and desirable properties
- That even if we take this problem apart, there is simply no ”better way” to define what makes a ”good” society
- This means that ”good” needs to be specified: *according to what* can we judge that a social state is better than another?
  - It is *efficient* **(Pareto criterion)**
  - It provides more *happiness* for the greatest number **(utilitarianism)**
  - It is *fair* regarding everyone's access to available goods **(egalitarian liberalism)**
  - What else? There are many other theories of ethics and justice that can inform us on how SWF should look like

WHAT NEXT?

In **session 4**, we have seen the problems with aggregating individual preferences and alternative normative criteria for evaluating social well-being

- But until now, we have not studied the implication of one's utility towards another's utility
- That is, we assumed so far that each individual's decision can be made without affecting others
- But how about if individuals' decision *do* affect others? Examples:
  - Smoking increases the utility of smokers but can decrease the utility of non-smokers
  - Playing music at 3:00AM can increase my own utility but also decrease the one of my neighbour who prefers to sleep
  - Pollution can increase some firms' profits, but can also decrease others'
  - Etc.

WHAT NEXT?

In **session 5**, we will study the concept of *externality*:

- When one individual cares directly about another individuals' production or consumption
- There can be two types of externalities:
  - *Negative*: it affects one in a bad way (e.g. a non-smoker surrounded by smokers)
  - *Positive*: it affects one in a good way (e.g. knowledge sharing)
- In particular, we will study how the market can take into account preferences that affect others

Intermediate Microeconomics

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